Introduction to Stress Analysis by the PhotoStress® Method

1.0 General Information

PhotoStress® is a widely used full-field technique for accurately measuring surface strains to determine the stresses in a part or structure during static or dynamic testing.

With the PhotoStress method, a special strain-sensitive plastic coating is first bonded to the test part. Then, as test or service loads are applied to the part, the coating is illuminated by polarized light from a reflection polariscope. When viewed through the polariscope, the coating displays the strains in a colorful, informative pattern which immediately reveals the overall strain distribution and pinpoints highly strain areas. With an optical transducer (compensator) attached to the polariscope, quantitative stress analysis can be quickly and easily performed. Permanent records of the overall strain distribution can be made by photography or by video recording.

With PhotoStress, you can . . .

- Instantly identify critical areas, highlighting overstressed and understressed regions.
- Accurately measure peak stresses and determine stress concentrations around holes, notches, fillets, and other potential failure sites.
- Optimize the stress distribution in parts and structures for minimum weight and maximum reliability.
- Measure principal stresses and directions at any point on the coated part.
- Test repeatedly under varying load conditions, without recoating the part.
- Make stress measurements in the laboratory or in the field — unaffected by humidity or time.
- Identify and measure assembly stresses and residual stresses.
- Detect yielding, and observe redistribution of strains in the plastic range of deformation.

PhotoStress coatings can be applied to the surface of virtually any test part regardless of its shape, size, or material composition. For coating complex shapes (see Figure 1), liquid plastic is cast on a flat-plate mold and allowed to partially polymerize. While still in a pliable state, the sheet is removed from the mold and formed by hand to the contours of the test part (shown below). When fully cured, the plastic coating is bonded in place with special reflective cement, and the part is then ready for testing. For plane surfaces, pre-manufactured flat sheets are cut to size and bonded directly to the test part. See Application Notes IB-221 and IB-223.

Figure 1. PhotoStress coating being contoured to the surface of a vehicle water pump casting.

PhotoStress has an established history of successful applications in virtually every field of manufacture and construction where stress analysis is employed, including: automotive—farm machinery—aircraft and aerospace—building construction—engines—pressure vessels—shipbuilding—office equipment—bridges—appliances—plus many others.

2.0 Polarized Light — Fundamentals

Light or luminous rays are electromagnetic vibrations similar to radio waves. An incandescent source emits radient energy which propagates in all directions and contains a whole spectrum of vibrations of different frequencies or wavelengths. A portion of this spectrum, wavelengths between 400 and 800 nm [15 and 30 × 10^{-6} in], is useful within the limits of human perception.

The vibration associated with light is perpendicular to the direction of propagation. A light source emits a train of waves containing vibrations in all perpendicular planes. However, by the introduction of a polarizing filter *P* (Figure 2 on page 2), only one component of these vibrations will be transmitted (that which is parallel to the privileges axis of

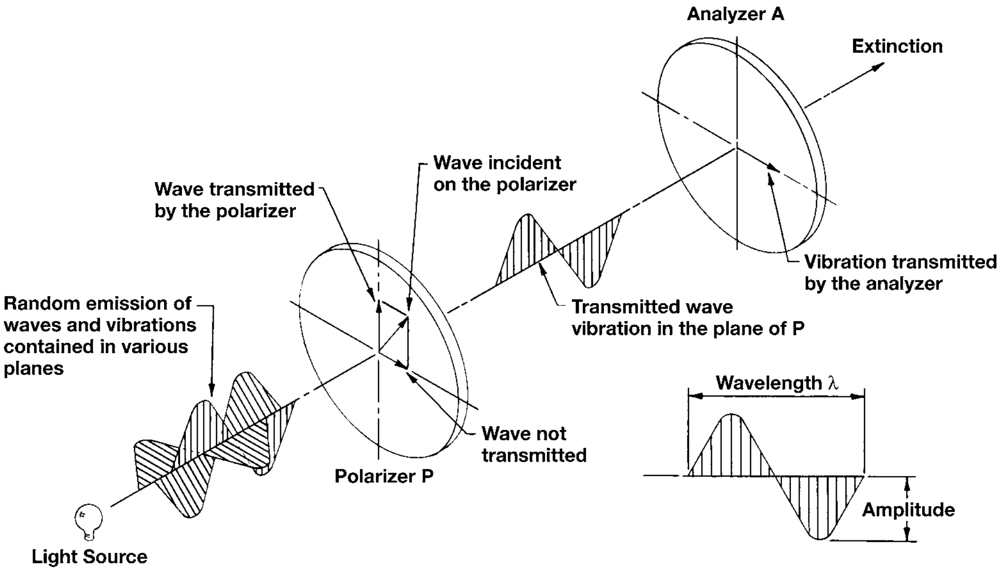

Figure 2. Polarization of light.

the filter). Such an organized beam is called polarized light or "plane polarized" because the vibration is contained in one plane. If another polarizing filter *A* is placed in its way, complete extinction of the beam can be obtained when the axes of the two filters are perpendicular to one another.

Light propagates in a vacuum or in air at a speed *C* of 3 × 10^{10} cm/sec. In other transparent bodies, the speed *V* is lower and the ratio *C/V* is called the index of refraction. In a homogeneous body, this index is constant regardless of the direction of propagation or plane of vibration. However, in crystals the index depends upon the orientation of vibration with respect to index axis. Certain materials, notably plastics behave isotropically when unstressed but become optically anisotropic when stressed. The change in index of refraction is a function of the resulting strain, analogous to the resistance change in a strain gage.

When a polarized beam a propagates through a transparent plastic of thickness *t*, where X and Y are the directions of principal strains at the point under consideration, the light vector splits and two polarized beams are propagated in planes X and Y (see Figure 3 on page 3). If the strain intensity along X and Y is ε_x and ε_y, and the speed of the light vibrating in these directions is V_x and V_y, respectively, the time necessary to cross the plate for each of them will be *t/V*, and the relative retardation between these two beams is:

$$\delta = C\left(\frac{t}{V_x} - \frac{t}{V_y}\right) = t\left(n_x - n_y\right) \qquad (1)$$

where: n = index of refraction

Brewster's law established that: “The relative change in index of refraction is proportional to the difference of principal strains”, or:

$$\left(n_x - n_y\right) = K\left(\varepsilon_x - \varepsilon_y\right) \qquad (2)$$

The constant *K* is called the “strain-optical coefficient” and characterizes a physical property of the material. It is a dimensionless constant usually established by calibration and may be considered similar to the “gage factor” of resistance strain gages. Combining the expressions above, we have:

$$\delta = tK\left(\varepsilon_x - \varepsilon_y\right) \quad \text{in transmission} \qquad (3)$$

$$\delta = 2tK\left(\varepsilon_x - \varepsilon_y\right) \quad \text{in reflection (light passes through the plastic twice)} \qquad (4)$$

Consequently, the basic relation for strain measurement using the PhotoStress (photoelastic coating) technique is:

$$\left(\varepsilon_x - \varepsilon_y\right) = \frac{\delta}{2tK} \qquad (5)$$

Due to the relative retardation δ, the two waves are no longer in phase when emerging from the plastic. The analyzer A will transmit only one component of each of these waves (that parallel to A) as shown in Figure 3. These waves will interfere and the resulting light intensity will be a function of:

- the retardation δ
- the angle between the analyzer and direction of principal strains ($\beta - \alpha$)

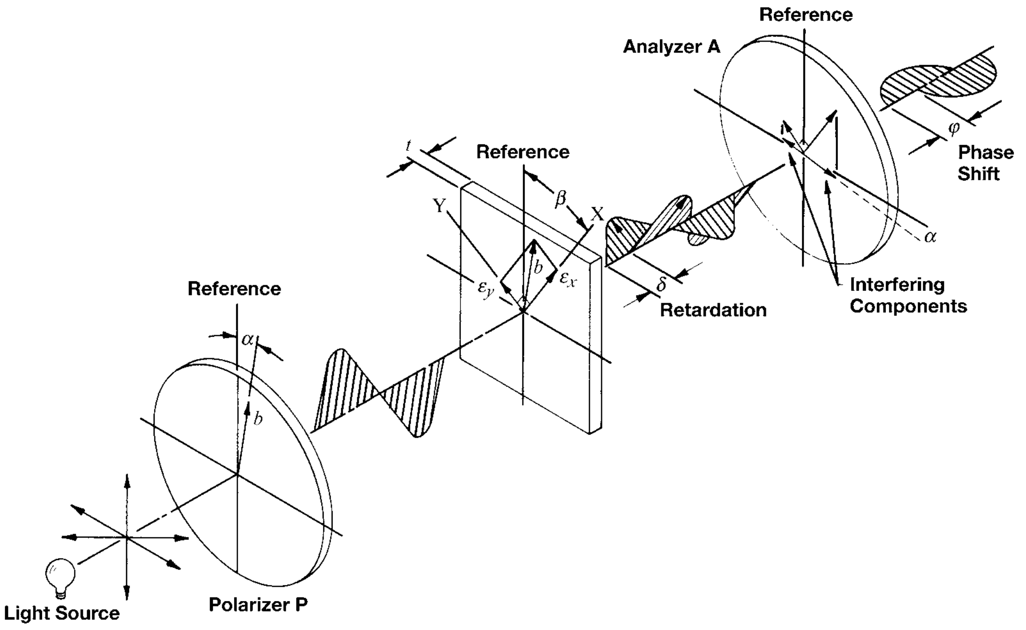

Figure 3. Plane polariscope.

In the case of plane polariscope, the intensity of light emerging will be:

$$I = b^2 \sin^2 2(\beta - \alpha) \sin^2 \frac{\pi\delta}{\lambda} \qquad (6)$$

The light intensity becomes zero when $\beta - \alpha = 0$, or when the crossed polarizer/analyzer is parallel to the direction of principal strains. Thus, a plane polariscope setup is used to measure the principal strain directions.

Adding optical filters known as quarter-wave plates in the path of light propagation produces circularly polarized light (Figure 4), and the image observed is not influenced by the direction of principal strains. The intensive of emerging light thus becomes:

$$I = b^2 \sin^2 \frac{\pi\delta}{\lambda} \qquad (7)$$

In a circular polariscope, the light intensity becomes zero when $\delta = 0$, $\delta = 1\lambda$, $\delta = 2\lambda$ . . . , or in general:

$$\delta = N\lambda$$

where N is 1, 2, 3, etc.

This number N is also called fringe order and expresses the size of δ. The wavelength selected is:

$$\lambda = 22.7 \times 10^{-6} \text{ in } [575 \text{ nm}]$$

The retardation, or photoelastic signal, is then simply described by N. As an example:

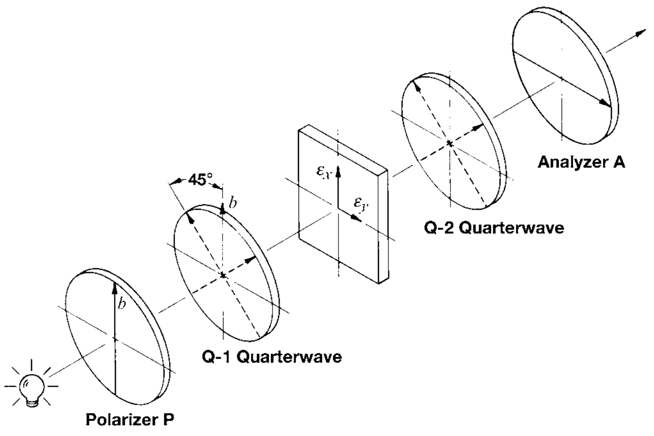

Figure 4. Circular polariscope.

If N = 2, (δ) Retardation = 2 Fringes
or $\delta = 2\lambda$
or $\delta = 45.4 \times 10^{-6}$ in [1150 nm]

Once $\delta = N\lambda$ is known, the principal strain difference is obtained by:

$$\varepsilon_x - \varepsilon_y = \frac{\delta}{2tK} = N\frac{\lambda}{2tK} = Nf \qquad (8)$$

where the *fringe value*, f, contains all constants, and N is the result of measurements.

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For background reference as needed, the topic of polarized light, as used in conjunction with photoelasticity, is treated more comprehensively in the textbooks and other references listed in the bibliography.

3.0 PhotoStress Instrumentation and Materials

3.1 Reflection Polariscope

For PhotoStress analysis, a reflection polariscope is used to observe and measure the surface strains on the photoelastically coated test part (Figure 5). The PhotoStress Plus LF/Z-2 Reflection Polariscope (Figure 6) covers a wide range of strain measurement capabilities. For instance, measurements made on small parts or in regions of high stress concentration are both easier and more accurate when zooming in with the digital video camera supplied with the polariscope. And the standard light source used for static measurements is readily replaceable with an optimal stroboscopic light accessory for cyclical dynamic measurements.

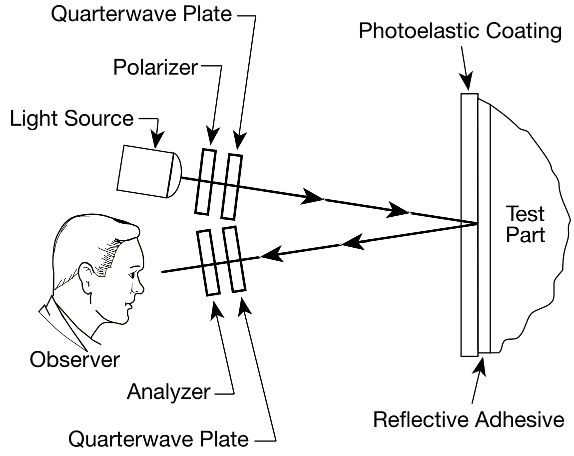

Figure 5. Schematic representation of reflection polariscope.

Figure 6. LF/Z-2 Reflection Polariscope.

3.2 Coating Materials

The selection of PhotoStress coatings and their proper application to the test part are most essential to the success of PhotoStress analysis. A wide range of coating materials (Figure 7) is available in both flat-sheet and liquid form for application to metals, concrete, plastics rubber, and most other materials. The coatings are carefully controlled formulations of resins blended to provide known and repeatable photoelastic properties, and are supplied with detailed application and handling instructions. Also available are specially designed application kits, containing everything required for successful installation of the PhotoStress coating on the test part.

Figure 7. PhotoStress coating materials: flat sheets, liquid plastics for casting contourable sheets, and adhesives.

4.0 Analysis of Photoelastic Fringe Patterns

PhotoStress offers the capability for the following types of analysis and measurement:

1. Full-field interpretation of fringe patterns, permitting overall assessment of nominal strain/stress magnitudes and gradients.
2. Quantitative measurements:
   a. The directions of principal strains and stresses at all points on the photoelastic coating.
   b. The magnitude and sign of the tangential stress along free (unloaded) boundaries, and in all regions where the state of stress is uniaxial.
   c. In a biaxial stress state, the magnitude and sign of the difference in principal strains and stresses at any selected point on the coated surface of the test object.

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4.1 Full-Field Interpretation of Strain Distribution

In addition to its capability for obtaining accurate strain measurements at preselected test points, PhotoStress provides another equally important capability to the stress analyst. This is the facility for immediate recognition of nominal strain (and stress) magnitudes, strain gradients, and overall strain distribution — including identification of overstressed and understressed areas. This extremely valuable attribute of PhotoStress, described as *full-field interpretation*, is unique to photoelastic methods of stress analysis. Its successful application depends only on the recognition of fringe orders by color, and an understanding of the relationship between fringe order and strain magnitude.

When a photoelastically coated test object is subjected to loads, the resulting stresses cause strains to exist generally throughout the part and over its surface. The surface stresses and strains are commonly the largest, and of the greatest importance. Because the PhotoStress coating is intimately and uniformly bonded to the surface of the part, the strains in the part are faithfully transmitted to the coating. The strains in the coating produce proportional optical effects which appear as *isochromatic* fringes when viewed with a reflection polariscope.

The PhotoStress fringe pattern is rich with information and insights for the design engineer. If, for example, a part is being stress analyzed as a result of field service failures, the overall PhotoStress pattern will usually suggest corrective measures for preventing the failures — often involving material removal and weight savings. Because of the full-field picture of stress distribution generated, it may be noted that the overstressed zone responsible for the failures is surrounded by an area of near-zero stress; and a slight change in shape will redistribute the stresses so as to eliminate the stress concentration, while forcing the understressed material to carry its share of the load.

Similarly, in prototype stress analysis for produce development purposes, the photoelastic pattern can point the way toward design modifications to achieve the minimum-weight, functionally adequate part — i.e., the optimum design. In addition, full-field observation of the stress distribution easily shows the effects of varying modes of loading, as well as the relative significance of individual loads, and/or load directions. These examples are merely indicative of the many ways in which full-field fringe patterns in PhotoStress coated test parts speak out to the knowledgeable stress analyst and provide a level of comprehension not achievable from “blind” strain measurements at a point.

4.2 Fringe Generation

When observed with a reflection polariscope, the PhotoStress fringe pattern appears as a series of successive and contiguous different-colored bands (isochromatics) in which each band represents a different degree of birefringence corresponding to the underlying strain in the test part. Thus, the color of each and uniquely identifies the birefringence, or fringe order (and strain level), everywhere along that band. With an understanding of the unvarying sequence in which the colors appear, the photoelastic fringe pattern can be read much like a topographical map to visualize the stress distribution over the surface of the coated test part.

Staring with the unloaded test part, and applying the load, or loads, in increments, fringes will appear first at the most highly stresses points (Figure 8). As the load is increased and new fringes appear, the earlier fringes are pushed toward the areas of lower stress. With further loading, additional fringes are generated in the highly stressed regions and move toward regions of zero or low stress until the maximum load is reached. The fringes can be assigned ordinal numbers (first, second, third, etc.) as they appear, and they will retain their individual identifies (“orders”) throughout the loading sequence. Not only are fringes *ordered* in the sense of serial numbering, but they are also *orderly* — i.e., they are continuous, they never cross or merge with one another, and they always maintain their respective positions in the ordered sequence.

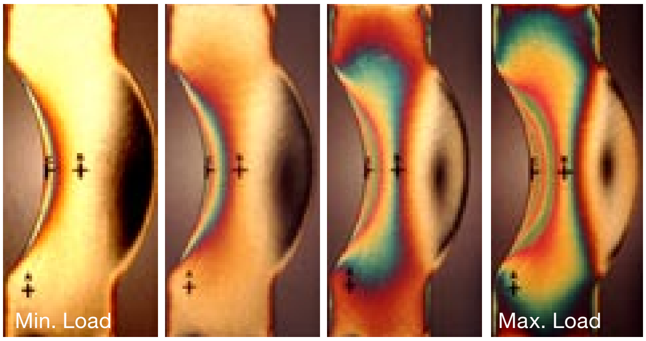


Figure 8. Incrementally loaded test part.

4.3 Fringe Identification

White light, generally used for full-field interpretation of fringe patterns in PhotoStress testing, is composed of all wavelengths in the visual spectrum. Thus, the relative retardation which causes extinction of one wavelength (color) does not generally extinguish others. When, with increasing birefringence, each color in the spectrum is extinguished in turn according to its wavelength (starting with violet, the shortest visible wavelength), the observer sees the complementary color. It is these complementary colors that make up the visible fringe pattern in white light. The complete color sequence is given in Table 1 (on page 6), including, for each color, the relative retardation and the numerical fringe order. Figure 9 (also on page 6) shows fringe identification on a test sample subjected to a uniaxial tension force. Because of simultaneous multiple extinction of colors, the higher order fringes become fainter than the first, and falls in the transition area between red and green. Fringe orders above 4 or 5 are not distinguishable by color in white light. Although fringe orders higher than 3 are rarely encountered (or needed) in stress analysis with PhotoStress coatings, fringes of very high order can always be detected by using the Model 036 Monochromator with the reflection polariscope.

A simple cantilever beam as shown in Figure 10 provides a means for understanding fringe identification as related to strain magnitude. The beam is coated on one side with PhotoStress plastic and clamped (coated side up) to the edge of a bench or table. A weight is hung, using a wire or cable, on the free end of the beam. When observed with the polariscope (circular light operation), the retardation increases proportionally to the strain.

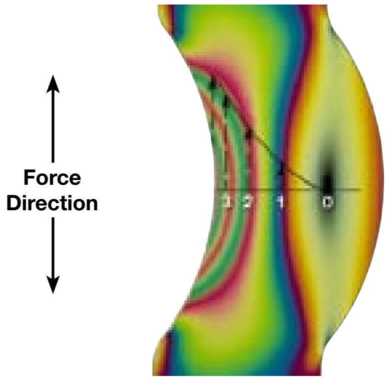


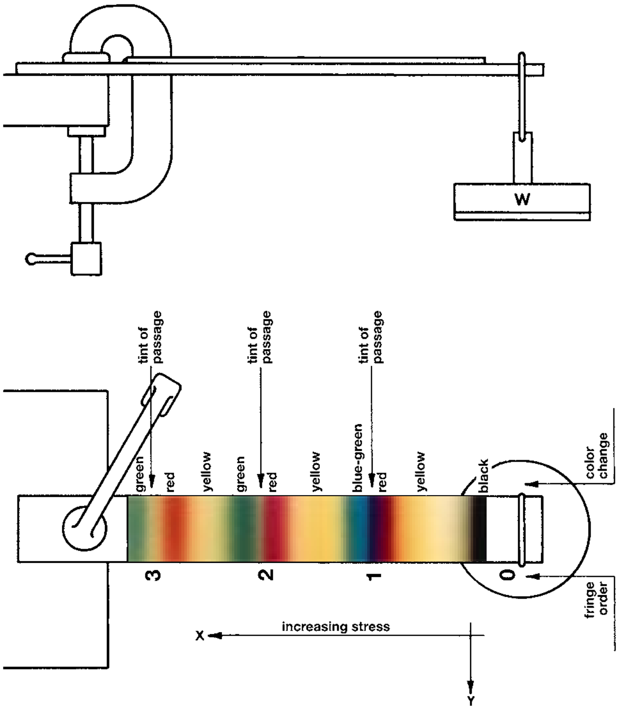
Figure 9. Strain field with fringes identified.

TABLE 1 — ISOCHROMATIC FRINGE CHARACTERISTICS

COLOR	APPROXIMATE RELATIVE RETARDATION nm	in × 10^{-6}	FRINGE ORDER N
Black	0	0	0
Pale Yellow	345	14	0.60
Dull Red	520	20	0.90
Red/Blue Transition	575	22.7	1.00
Blue-Green	700	28	1.22
Yellow	800	32	1.39
Rose Red	1050	42	1.82
Red/Green Transition	1150	45.4	2.00
Green	1350	53	2.35
Yellow	1440	57	2.50
Red	1520	60	2.65
Red/Green Transition	1730	68	3.00
Green	1800	71	3.10

4.4 Quantitative Significance of Fringes

Photoelastic fringes have characteristic behaviors which are very helpful in fringe pattern interpretation. For instance, the fringes are ordinarily continuous bands, forming either closed loops or curved lines. The black zero-order fringes are usually isolated spots, lines, or areas surrounded by or



The fringes are related to increasing strain magnitude as summarized in the table. (See Relationships Between Fringe Orders and Magnitudes of Strain and Stress on page 8. For reference, see Eq. (8) on page 3]:

$$\left.\begin{array}{l} t = 0.100 \text{ [2.54 mm]} \\ K = 0.15 \\ \lambda = 22.7 \times 10^{-6} \text{ in [575 nm]} \end{array}\right\} \quad f = 757\ \mu\text{in/in/fringe } [\mu\text{m/m/fringe}]$$

Fringe Order, N	Strain, $(\varepsilon_x - \varepsilon_y) = Nf$
0 (Black Fringe)	0
1 (Red-Blue)	757 μin/in [μm/m] ($1f$)
2 (1st Red-Green)	1514 μin/in [μm/m] ($2f$)
3 (2nd Red-Green)	2271 μin/in [μm/m] ($3f$)

Figure 10. PhotoStress analysis of cantilever beam.

adjacent to higher-order fringes. The fringes never intersect, or otherwise lose their identifies, and therefore the fringe order and strain level are uniform at every point on a fringe. Furthermore, the fringes always exist in a continuous sequence by both number and color. In other words, if the first- and third-order fringes are identified, the second-order fringe must lie between them. The color sequence in any direction establishes whether the fringe order and strain level increase or decrease in that direction.

It turns out that the characteristics of photoelastic fringes are the same as those of constant-level contours on a colored topographic map. As a result, of any photoelastic pattern can be considered, and visualized, as a contour map of the difference (without regard to sign) between principal strains or stresses over the surface of the test part. In other words, the magnitudes of the strain levels, as indicated by the fringe orders, correspond directly to constant-altitude levels on a topographic map. And the fringe pattern depicts peaks and valleys, plains and mesas — with "sea level" represented by the zero-order fringes.

If there is a zero-order fringe in the field of view, it will usually be obvious by its black color. Assuming that the coated test part has a free square corner or pointed projection, the stress there will always be zero, and a zero-order fringe (spot) will exist in the corner, irrespective of the load magnitude, but shrinking in size slightly as the load increases. When there is no zero-order fringe evident, the first-order fringe can often be recognized because of the bright colors adjacent to the purple tint of passage. As an alternative, when the test object can be loaded incrementally from an initially stress-free state, the starting zero-order fringe which covers the entire coating can usually be followed throughout the loading process as it recedes toward unstressed points, and regions where the difference in principal stresses is zero.

Once one fringe has been identified, orders can be assigned to the other fringes, making certain that the direction of increasing fringe order corresponds to the correct color sequence — i.e., yellow-red-green, etc. By this process the observer can quickly locate the highest fringe orders and, generally, the most highly strained regions. Areas of closely spaced fine fringes will usually attract the observer's attention, since regions of steep strain gradient ordinarily signify high strains as well. The stress analyst will also note any large areas where the pattern is almost uniformly black or gray, usually indicating a significantly understressed region.

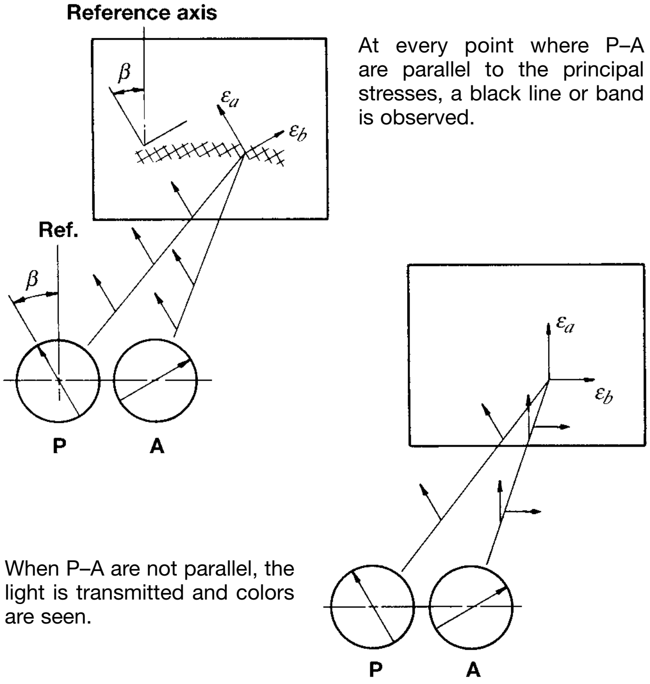
Frequently, the process of locating the highest fringe orders will lead the observer to one or more critical points on a free boundary. When this occurs, the stress analyst knows that the non zero principal stress at such a point is tangent to the boundary, and its magnitude can be obtained directly by multiplying the fringe order by a constant. The sign of the stress, plus or minus for tension or compression, can also be determined very easily on a free boundary with the reflection polariscope.

5.0 Measurement of Principal Strain Directions

5.1 Measurement Principal

The principal strain directions are always measured with reference to an established line, axis, or plane. Therefore, the initial step for the determination of the direction or principal strains (or stresses) will be to select a convenient reference. In most cases, the reference direction is suggested immediately, like an axis of symmetry of the test part or structure; in other cases, a vertical or horizontal line will suffice.

When a plane-polarized beam of light transverses a photoelastic coating on a part subjected to stress, it splits into waves propagating at different speeds along the direction of the principal strains. After emerging from the plastic, these two waves will be out of phase with one another and will not recombine into a single vibration parallel to the one entering the plastic. However, at points where the direction of the principal stresses are parallel to the axis of the polarizing filter, the beam will be unaffected and the emerging vibration will be parallel to the entering vibration. An analyzing filter *A* with its axis perpendicular to the polarizing filter *P* will reproduce extinction of the vibrations at these points (see Figure 11).



At every point where P–A are parallel to the principal stresses, a black line or band is observed.

When P–A are not parallel, the light is transmitted and colors are seen.

Figure 11. Principal stress directions revealed by rotation of the polarizer/analyzer axes to produce complete extinction of light at the test point.

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Observing the stresses part through a reflection polariscope (Figure 12), black lines (or even areas) appear. These lines are called *isoclinics*. At every point on an isoclinic, the directions of principal strains are parallel to the direction of polarization of A and P. With respect to the selected reference axis, the measurement of directions at a point is simply accomplished by the rotation of A and P together until a black isoclinic appears at the point where the directions are to be measured.

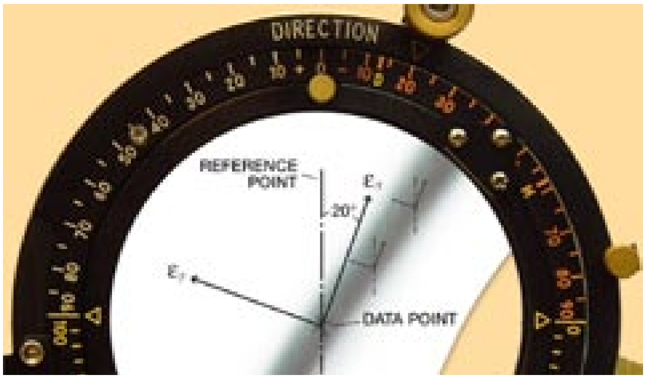

Figure 12. Principal stress directions revealed by rotation of the polarizer/analyzer axes to produce complete extinction of light at the test point, a plane polariscope setup is used. Then, an isoclinic is positioned over the point and the directions measured with respect to an established reference. Note, with the Model LF/Z-2 Polariscope, a vertical axis is chosen and the principal strain directions are read from the calibrated dial.

If the isoclinics are narrow and sharply defined, it means that the directions of ε_x and ε_y vary rapidly from one location to the next. Isoclinics forming broad black lines or areas indicate that the ε_x and ε_y directions vary slowly in that region. When this occurs, the boundary surrounding the entire isoclinic should be marked (not merely the center). In the case of a tensile specimen with a constant cross section, an isoclinic will be seen to cover the entire area when the axes of polarization coincide with the specimen axes, wince the direction of ε_x is the same at every point.

To aid in identifying the strain directions, a laser light is attached to the LF/Z-2 Reflection Polariscope. When turned on, a "laser line" is projected on the PhotoStress coated part showing the principal strain directions at the point of measurement.

6.0 Measurement of Stress and Strain Magnitudes

6.1 Relationships Between Fringe Orders and Magnitudes of Strain and Stress

The fringe orders observed in PhotoStress coatings are proportional to the difference between the principal strains in the coating (and in the surface of the test part). This simple linear relationship is expressed as follows [repeating Equation (8) her for convenient reference]:

$$\varepsilon_x - \varepsilon_y = Nf \qquad (8)$$

where: $\varepsilon_x - \varepsilon_y$ = principal strains; N = fringe order

$f =$ (fringe value of coating)

λ = wavelength (in white light, 22.7×10^{-6} in or 575 nm

t = thickness of coating

K = strain optical coefficient of coating

Equation (8) can also be written in terms of shear strain, γ_{xy}:

$$\gamma_{xy} = Nf \qquad (9)$$

where: γ_{xy} = maximum shear strain (in the plane of the part surface at any point

The significance of the preceding is that the difference in the principal strains, or the maximum shear strain in the surface of the test part, can be obtained by simply recognizing the fringe order and multiplying by the fringe value of the coating.

Engineers and designers often work with stress rather than strain; and, for this purpose, Eqs. (8) and (9) can be transformed by introducing Hooke's law for the biaxial stress state in mechanically isotropic materials:

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) \qquad (10)$$

$$\sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) \qquad (11)$$

and,

$$\sigma_x - \sigma_y = \frac{E}{1+\nu}\left(\varepsilon_x - \varepsilon_y\right)^* \qquad (12)$$

Substituting Equation (8) into Equation (12),

$$\sigma_x - \sigma_y = \frac{E}{1+\nu}Nf \qquad (13)$$

where: $\sigma_x - \sigma_y$ = principal stresses in test part surface

E = elastic modulus of test part

ν = Poisson's ratio of test part

And, noting that the maximum shear stress, τ_{MAX}, *in the plane of the surface* at any point is $(\sigma_x - \sigma_y)/2$,

$$\tau_{MAX} = \frac{1}{2}\left(\frac{E}{1+\nu}\right)Nf \qquad (14)$$

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Equations (8) and (13), which are the primary relationships used in photoelastic coating stress analysis, give only the *difference* in principal strains and stresses, not the individual quantities. To determine the individual magnitudes and signs of either the principal strains or stresses generally requires, for biaxial stress states, a second measurement, such as the sum of the principal strains (see page 12). There are many cases, however, when these equations provide all of the information needed for stress analysis. For instances, when the ratio of principal stresses can be inferred from other considerations — a uniform shaft in torsion ($\sigma_x - \sigma_y = -1$), a thin walled pressure vessel (σ_x / σ_y) = 2), etc. — this relationship can be combined with Equation (13) to solve for the individual principal stresses. And, whenever the stress state is known to be uniaxial, with either σ_x or σ_y being zero, there is only one nonzero principal stress in the plane of the test-part surface, and this can be obtained directly from Equation (6). For example, if $\sigma_y = 0$,

$$\sigma_x = \frac{E}{1+\nu} Nf \qquad (15)$$

The cases in which one of the principal surface stresses is zero include all straight, uniform-cross-section members in axial tension or compression (and bending), away from points of load application. Even for mildly tapered members, so loaded, the stress state is very nearly uniaxial, and Equation (15) can often be applied as a very good approximation. A much more important class of cases from the viewpoint of practical stress analysis involves all points on the boundaries and free edges of the test part.

Consider, for example, an unloaded hole penetrating the test part. At every point on the edge of the hole the principal axes are normal and tangential, respectively, to the edge. Because the principal stress normal to the edge is necessarily zero, the stress state is uniaxial, and the only nonzero principal stress is everywhere tangent to the hole edge. There are many other cases, such as projecting flanges and ribs, and "two-dimensional" objects in general, in which the stress state on the unloaded edge is always uniaxial. For all such cases, the single nonzero principal stress, which is tangent to the edge, can be determined directly from the observed fringe order by substituting into Equation (15); or, in effect, multiplying the fringe order by a constant.

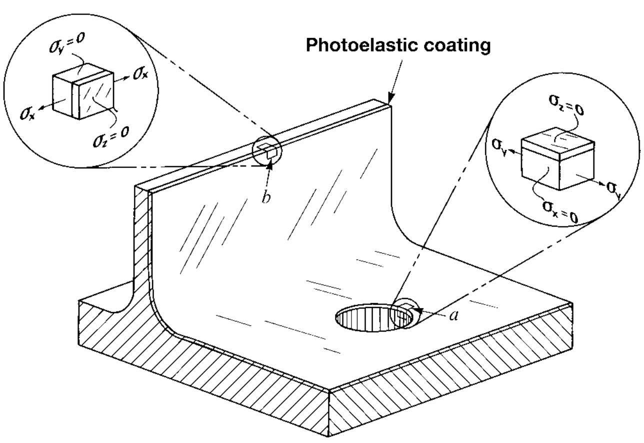

Figure 13. Section from a coated test member showing uniaxial stress states on free edges.

Figure 13 shows a portion of the surface of a steel machine part to which a PhotoStress coating has been applied. As indicated, the coating has been finished to accurately match the edge of the hole and that of the rib. The uniaxial stress state at points *a* and *b* is demonstrated by the enlarged free-body diagrams of elements of matter removed from the edges for examination. With the part under normal service loading, and viewing the coating with the reflection polariscope, a fringe order of 2 is observed at point *a*, and about 3/4 at point *b*. Previous calibration has established a fringe value of 1100$\mu\varepsilon$ per fringe for this coating. Thus, the stress at point a in the more critical region can be calculated directly from Equation (15) assuming (for steel) that $E/(1 + \nu) = 23.5 \times 10^6$ in [162 GPa]:

$$\sigma_x = 23.5 \times 10^6 \times 1100 \times 10^{-6} \times 2 = 51\,700 \text{ psi or,}$$

$$\sigma_x = 162 \times 10^9 \times 1100 \times 10^{-6} \times 2 = 356 \text{ MPa}$$

And similarly, the stress at the edge of the rib is about 19 400 psi [134 MPa].

Summarizing, the difference between principal strains can be determined from Equation (8), and the difference between principal stresses from Equation (13), at any point on a photoelastically coated surface. At points where the stress state is uniaxial, Equation (15) gives the principal stress in each case, the result is obtained by multiplying the observed fringe order by a constant. It remains, then, only to identify the fringe order at the point of measurement. Techniques for accomplishing this precisely and positively with the Model LF/Z-2 Reflection Polariscope follow.

6.2 Measurements at a Point

It has been shown that in the first step of measurement one observes the whole area and assigns to each fringe its order (N = 1, 2, 3, etc.) At every point on a fringe, N is then known and therefore:

$$\varepsilon_x - \varepsilon_y = Nf$$

In general, the point of interest on the structure will fall between fringes, and it will be necessary to establish the

---

*It should be kept in mind that the biaxial Hooke's law, involved in Equations (1) - (15), is strictly applicable **only to homogeneous materials which are isotropic and linear-elastic in their mechanical properties.**

"fractional order" or fraction of a fringe. The technique used is called "compensation" which is accomplished using the PhotoStress Plus Model 832 "Null-Balance" Compensator.

6.2.1 Measurements Using The Null-Balance Compensation Method

Null-balance compensation operates on the principal of introducing into the light path of the polariscope a calibrated variable birefringence of opposite sign to that induced in the photoelastic coating by the strain field. When the opposite-sign variable birefringence is adjusted to precisely match the magnitude of the strain-induced birefringence, complete cancellation will occur, and the net birefringence in the light path will be zero. The condition of zero net birefringence is easily recognized because it produces a black fringe in the isochromatic pattern where, before introducing the compensating birefringence, a colored fringe existed (Figure 14). The device for synthesizing a calibrated variable birefringence is known as a *null-balance compensator*.

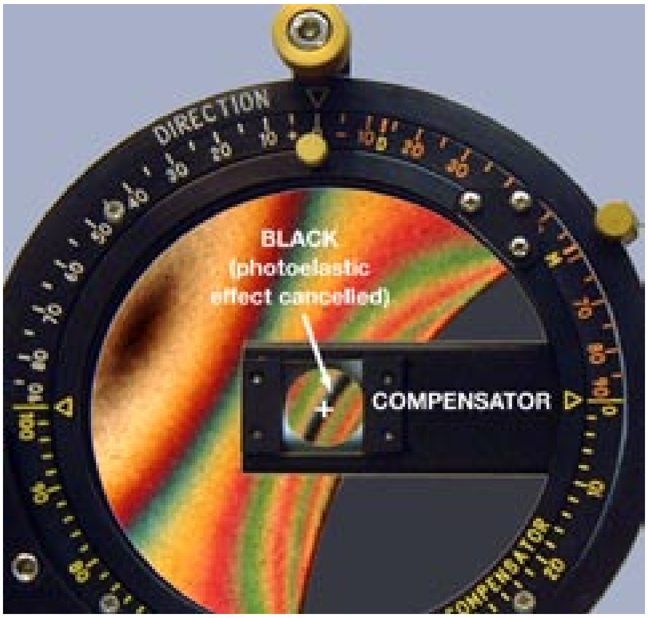

Figure 14. Initially colored fringe is rendered black by null-balance compensation.

The manner in which a null-balance compensator operates is illustrated schematically in Figure 15 by analogy with the common knife-edge balance. The strain-induced birefringence (or optical "signal") is represented by an unknown mass on the left-hand pan of the scale, where it produces a counterclockwise moment, tipping the pointer off from center to the left. Known masses can be placed on the right-hand pan (introducing a clockwise moment) until the pointer is brought back to center again. When the pointer is centered, the sum of the known calibrated masses equals the unknown mass. The operation of the compensator directly parallels that of the balance — that is, compensating birefringence is added to the light path until is exactly balances the birefringence induced in the coating by the strain field on the surface of the coated part.

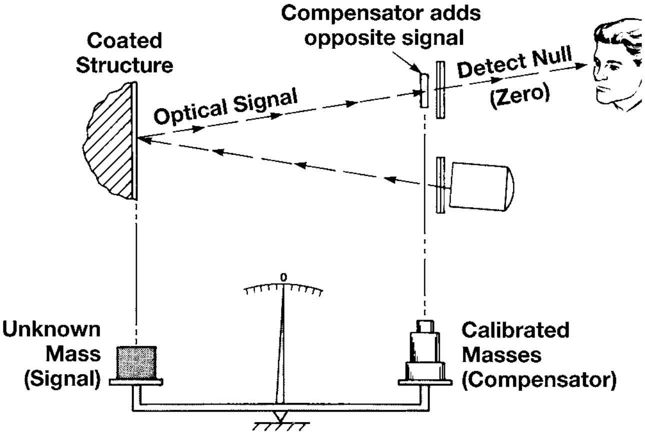

Figure 15. Principal of null-balance compensation.

With the Model 832 Electronic Compensator and PSCalc computer software (Figures 16a and 16b on page 11), measurement and calculation of the strain/stress values is simple and quick. At the point of measurement, an initial no-load (R_O) reading is made with the compensator. A second reading (R_{LOAD}) is then made after loading the part. After these null-balance readings (making the point of measurement black), the numerical information is electronically transferred to a computer configured with PhotoStress PSCalc software. The computer instantly makes and displays the strain/stress calculations at the selected points of measurement. Prior to making the compensator readings, other information regarding the type and thickness of the PhotoStress coating being used, physical constants of the material being tested, the test load sequence, etc., is entered into the software.

In order for the null-balance measurement to be achieved, the compensator must first be aligned in the direction of the algebraically maximum principal strain. This is easily determined by establishing the directions of the two principal strains at the point of measurement with an isoclinic measurement as described on page 8. The compensator is then aligned with one of these directions, and the compensation attempted. If null-balance cannot be achieved, it means the compensator is aligned with the minimum principal strain direction. Repositioning the compensator 90° away will allow the null-balance compensation to be performed.

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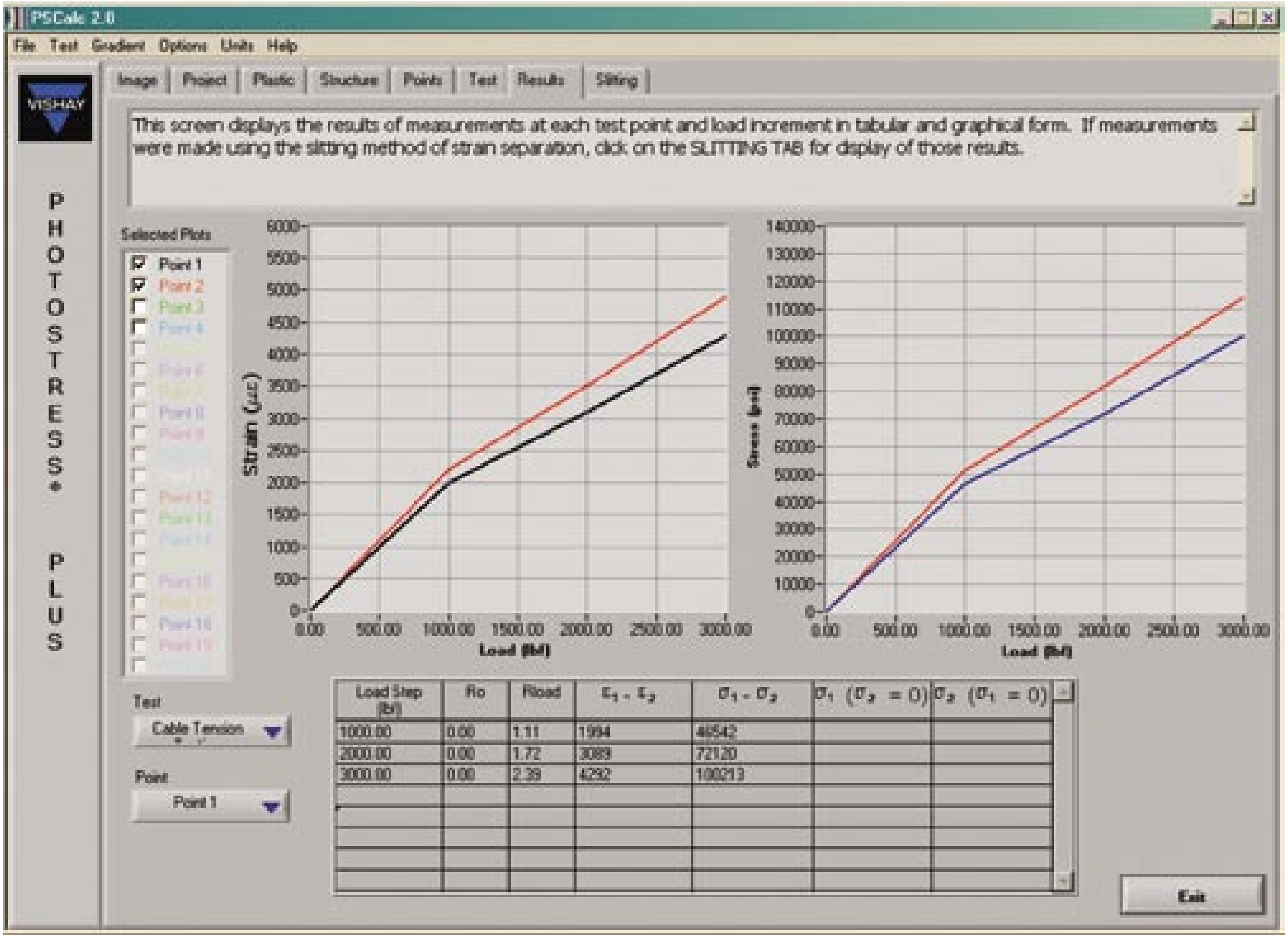
7.0 Principal Strain/Stress Separation Methods

In order to obtain principal strain/stress values at locations removed from free boundaries, an additional measurement is required. This is usually obtained by using PhotoStress Separator Gages or experimentally by creating an artificial free boundary called a slit. The pages that follow will provide an introduction to each method and the governing equations associated with them.

7.1 Strain Gage Separator Method

If the sum of the principal strains can be determined at the same point where the difference of the principal strains is measured, then the separate principal strain values are obtainable by simply solving equations simultaneously.

Figure 16a. Model 832 Compensator.



This screen displays the results of measurements at each test point and load increment in tabular and graphical form. If measurements were made using the slitting method of strain separation, click on the SLITTING TAB for display of those results.

Load Step (lbf)	Ro	Rload	$\varepsilon_1 - \varepsilon_2$	$\sigma_1 - \sigma_2$	$\sigma_1\ (\sigma_2 = 0)$	$\sigma_2\ (\sigma_1 = 0)$
1000.00	0.00	1.11	1994	46542		
2000.00	0.00	1.72	3089	72120		
3000.00	0.00	2.39	4292	100213		

Figure 16b. PSCalc® Software results.

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The PhotoStress Separator Gage is based on this fundamental principle of mechanics. As shown in Figure 17, the gage grid consists of two perpendicular elements connected in series. The indicated strain from the gage then corresponds to $(\varepsilon_x + \varepsilon_y)/2$ regardless of the gage orientation on the coated test surface. Representing the gage output signal by the symbol S_G for convenience in algebraic manipulation,

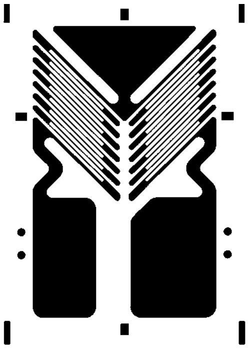

Figure 17. PhotoStress Separator Gage.

$$S_G = \frac{\varepsilon_x + \varepsilon_y}{2} \tag{16}$$

and $\varepsilon_x + \varepsilon_y = 2S_G$

Adding to, and subtracting from, the measurement of the difference of principal strains,

$$\varepsilon_x - \varepsilon_y = N_N f$$
$$\varepsilon_x + \varepsilon_y = 2S_G$$
$$\varepsilon_x = S_G + \frac{N_N f}{2} \tag{17}$$

and

$$\varepsilon_x - \varepsilon_y = N_N f$$
$$-\varepsilon_x - \varepsilon_y = -2S_G$$
$$\varepsilon_y = S_G - \frac{N_N f}{2} \tag{18}$$

In practical applications, the usual procedure is to first complete all PhotoStress observations and normal-incidence measurements (N_N) on the coated test object. Following this, separator gages are installed on the coating at the potentially critical points established by PhotoStress analysis. Loads are then reapplied to the test object, and the separator gage measurements are recorded.

The PhotoStress Separator Gage (for use on high-modulus coatings only) embodies a number of special features designed for ease of use and optimum performance in PhotoStress applications. First in importance, of course, is that the gage does not require any particular angular orientation. It is simply bonded at the point where separation measurements are desired. Preattached leadwires are provided to avoid the problems that users may have in soldering the leads to the gage before installation, or attempting to do so after the gage is bonded to the photoelastic coatings. The gage grid is also encapsulated in polyimide to eliminate the need for a protective coating in most PhotoStress applications.

Grid resistance of the separator gage is 200 ohms; and it is intended that the gage be connected to an appropriate Vishay strain indicator (Figure 18).

Figure 18. Model P3 Strain Indicator and Recorder for displaying $\varepsilon_x + \varepsilon_y/2$.

The complete technical background on the separator gage and its application can be found in Tech Note TN-708.

7.2 Slitting Method

A slit made in the PhotoStress coating in the direction of ε_1 or ε_2, creates a uniaxial stress state at the slit boundary. To create a slit, appropriate tooling is needed, such as the slitting tool shown in Figure 19.

Figure 19. A slitting tool.

Introduction to Stress Analysis by the PhotoStress® Method

To measure the strains at a slit boundary, refer to the equations below:

where: $\varepsilon_1, \varepsilon_2$ = principal strains

σ_1, σ_2 = principal stresses

E_s = elastic modulus of structure

ν_s = Poisson's constant of structure

ν_c = Poisson's constant of PhotoStress Coating Material

f_c = calibration value for PhotoStress Coating Material

R_o = Digital compensator reading (no load)

R_1 = Digital compensator reading (under load)

7.2.1 Slitting in the direction of ε_1:

Measurement before slitting

$$\varepsilon_1 - \varepsilon_2 = f_c \bullet (R_1 - R_0) \tag{19}$$

Measurement after slitting with compensator parallel to slit

$$\varepsilon_1 = \frac{f_c}{1+\nu_c} \bullet (R_1 - R_0) \tag{20}$$

Measurement after slitting with compensator perpendicular to slit

$$\varepsilon_1 = -\left[\frac{f_c}{1+\nu_c} \bullet (R_1 - R_0)\right] \tag{20a}$$

Calculated from (19) and (20)

$$\varepsilon_2 = \varepsilon_1 - \left[f_c \bullet (R_1 - R_0)\right] \tag{21}$$

Note that the only difference between Eqs. (20) and (20a) is that (20a) has a negative sign.

7.2.2 Slitting in the direction of ε_2:

$$\varepsilon_1 - \varepsilon_2 = f_c \bullet (R_1 - R_0) \qquad \text{the same as (19) above}$$

Measurement after slitting with Compensator parallel to slit

$$\varepsilon_2 = \frac{f_c}{1+\nu_c} \bullet (R_1 - R_0) \tag{22}$$

Measurement after slitting with Compensator perpendicular to slit

$$\varepsilon_2 = -\left[\frac{f_c}{1+\nu_c} \bullet (R_1 - R_0)\right] \tag{22a}$$

Calculated from (19) and (22)

$$\varepsilon_1 = \varepsilon_2 + \left[f_c \bullet (R_1 - R_0)\right] \tag{23}$$

Note that the only difference between Equations (22) and (22a) is that (22a) has a negative sign.

Once the principal strains have been measured, the principal stresses can then be calculated.

Introduction to Stress Analysis by the PhotoStress® Method

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Tech Note TN-704, “How to Select PhotoStress® Coatings”.

Tech Note TN-706, “Corrections to PhotoStress® Coating Fringe-Order Measurements”.

Tech Note TN-708, “Principal Stress Separation in PhotoStress® Measurements”.

Application Note IB-221, “Instructions for Coating and Contouring PhotoStress® Sheets.”

Application Note IB-223, “Instructions for Bonding Flat and Contoured PhotoStress® Sheets to Test-Part Surfaces”.

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